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Lasing on scar modes in fully chaotic microcavities

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Scar wave functions in a fully chaotic cavity are obtained numerically by an extended Fox-Li method. Lasing on the scar modes are observed in a semiconductor microcavity with a selective excitation of different scars controlled by corresponding shape of electrodes for current injection.

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Conventional lasers are built with cavity shapes which are integrable—with a one to one correspondence between modes and periodic or quasiperiodic motions due to the separability of equations of motion in different coordinates. Recently, lasing has been observed in small cavities with a variety of shapes—dye drops, glass microspheres, and semiconductor microcavities. Such shapes can be nonintegrable and chaotic, in the sense that the motions are not separable and the ray trajectories obey chaotic dynamics. The problem of modes of these lasers, and the correspondence between ray trajectories and optical modes is related to the problem of quantum chaos and fundamental issues in statistical mechanics [1,2]. In a fully chaotic cavity, there are no stable periodic orbits (SPO's), and each wave function, in principle, ''corresponds'' in a complex way to an infinite number of unstable periodic orbits (UPO's) [1]. One of the key issues of wave functions in chaotic cavities, or wave chaos, is scar wave functions, corresponding to wave functions which seem to localize on short UPO's [3].

Existence of scars is an important issue for the foundations of statistical mechanics. Ergodicity is the basis for the statistical mechanics [4,5]. The property of quantum mechanics corresponding to ergodicity is also important, and quantum ergodicity has recently been defined [2,6,7]. According to the correspondence principle, the ergodic properties of classical and quantum mechanics should correspond in the so-called semiclassical region of very high energies. Indeed, it has been exactly proven that most of the eigenfunctions of quantized classical-mechanical ergodic systems converge to uniform distributions [6–8]. Therefore, most of the eigenfunctions of quantized classical-mechanical chaotic systems distribute uniformly.

However, some of the eigenfunctions of the quantized strongly chaotic classical dynamical systems have been numerically found to localize on UPO's even in higher energy region [3,9]. Therefore, such localization is called ''scars,'' which means the scars in a uniform distribution of strongly chaotic systems [3]. The mechanism of the existence of scars has not yet been clarified completely, although several theoretical approaches have been made [3,10,11]. The evidence of scars in the actual experiments have also been found in the experiments of tunneling current of the quantum well, the microwave billiards, and the microcavity lasers [12–16].

The problem of scars can be discussed by the relation between ray and wave as well as classical and quantum mechanics because stationary solutions of the Maxwell equation and the Schrödinger equation are given by the same equation, i.e., the Helmholtz equation. Indeed, lasers of asymmetric resonant cavities have been studied and laser action described in terms of chaotic ray dynamics [17,18]. The laser diodes of two-dimensional resonant cavity provide an effective stage for investigating scars [15,16].

In this paper, we present the scars of strongly chaotic optical resonators belonging to a class that has been proven exactly to be fully chaotic. We also report on controlling of laser action on scar modes by selective pumping of those modes. The scar wave functions are numerically obtained by an extended Fox-Li mode calculation method, and also observed in experiments on a quantum-well laser diode. In this type of laser, localization of the light on scars can be detected in far-field emission patterns as beams corresponding to propagation along UPO's in the cavity.

The semiconductor laser diodes we investigated have the same shapes as so-called ''unstable optical resonators'' and ''concentric optical resonators,'' i.e., the radii of the circular mirrors on the edge are smaller than and equal to half of the cavity length, respectively, as shown in Fig. 1 [19]. The difference from the usual unstable and concentric resonators is that they have extra flat mirrors on the side walls, which makes closed resonators surrounded by mirrors. We call these resonators as ''closed unstable resonators'' and ''closed concentric resonators.'' Therefore, light reflects on the side

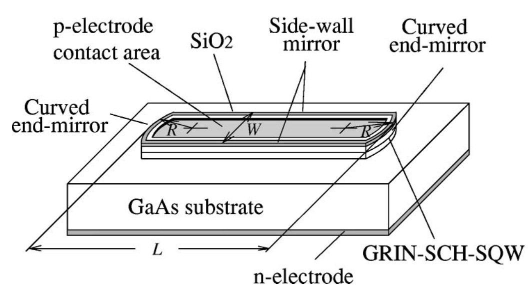

FIG. 1. Schematic illustration of the microcavity laser diode of closed unstable and concentric resonators. The shape of curved end mirrors are the arc of a circle. When the radius R of the curvature of the curved end mirrors is equal to or smaller than the cavity length L, the ray-dynamical trajectories are exactly proven to be chaotic.

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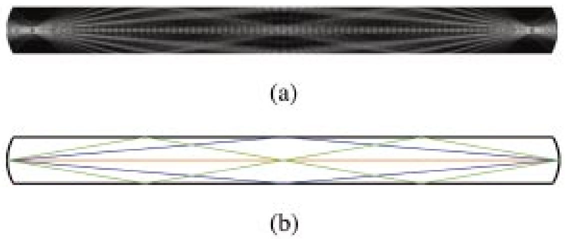

FIG. 2. (Color) The scar in a closed unstable resonator. (a) The eigenmode of the least loss. (b) Three kinds of periodic orbits. The red, blue, and green lines correspond, respevtively, to the closed trajetories of the bounce numbers 2, 4, 6 with the flat and curved mirrors.

walls as well as on the circular mirrors, and it is easy to imagine that the ray-dynamical trajectories are complicated due to multiple scattering inside the resonators.

Indeed, Bunimovich proved exactly the full chaoticity of the ray dynamics in the resonators of these shapes [20]. Accordingly, these resonators are totally different from those asymmetric resonant cavities deformed from a circular one in the sense that ray mechanics shows ''full chaos'' [17,18]. In fully chaotic dynamical systems, there exist no SPO's. The mathematical proof on full chaos of the ray mechanics is crucially important because SPO's could exist in arbitrarily small regions in the whole phase space even if the phase space structure of numerical simulations shows no such orbits.

The resonant cavities we discuss in this paper have a long waveguide in comparison to their width. Therefore, the quasistationary eigenfunctions corresponding to the resonances can be obtained by the extended version of the Fox-Li mode calculation method for the cavities of the flat mirrors on the side walls [19,21,22]. Assuming in the Helmholtz-Kirchhoff integral equation, the Neuman boundary condition of vanishing derivative of the wave function on the mirrors, and neglecting the reflecting waves from the circular mirror to the same circular mirror and those waves which violate the critical angle condition at the flat mirrors on the side walls yield the following iterative equations for the traveling waves from the left (right) circular mirror to the right (left) circular mirror:

$$E_{r(l)}(\boldsymbol{r}) = \int_A K(\boldsymbol{r},\boldsymbol{r}')E_{l(r)}(\boldsymbol{r}'), \tag{1}$$

where $E_{r(l)}(\boldsymbol{r})$ is the light field at the point $\boldsymbol{r}$ on the right (left) circular mirror. Here

$$K(\boldsymbol{r},\boldsymbol{r}') = -\frac{i}{2}H_1^{(2)}(nk|\boldsymbol{r}-\boldsymbol{r}'|), \tag{2}$$

where $H_1^{(2)}$ is the first order Hankel function of the second kind, while n and k are the refractive index and the wave number, respectively. In Eq. (1), the integrating area A is approximated by the original circular mirror and its mirror images above and below which express the reflections from the side walls only satisfy the critical angle conditions.

The round-trip calculations are repeated with an initial field distribution until the field distribution converges to the resonator eigenmode, i.e., the quasistationary eigenfunction. The power coupling coefficients are defined as the power of the absolute values of the complex eigenvalues of one round-trip calculation that correspond to the loss or decay rate of the resonator eigenmode [19,21,22].

In order to investigate the laser action on this scar mode, we have actually fabricated the closed unstable resonators by using a molecular-beam-epitaxially grown gradient-index, separate-confinement-heterostructure, single-quantum-well $GaAs/Al_xGa_{1-x}As$ structure as shown schematically in Fig. 1. The shapes of the circular and flat edges were formed by the reactive-ion-etching technique in order to realize extreme smoothness and verticality for our purpose of the investigation of the morphological effects of the resonant cavities on the lasing modes.

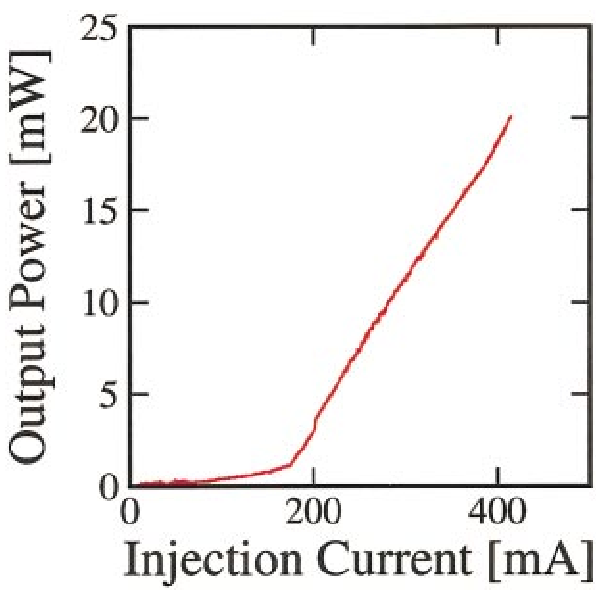

FIG. 3. (Color) Light output power versus injection current characteristics of the closed unstable resonator.

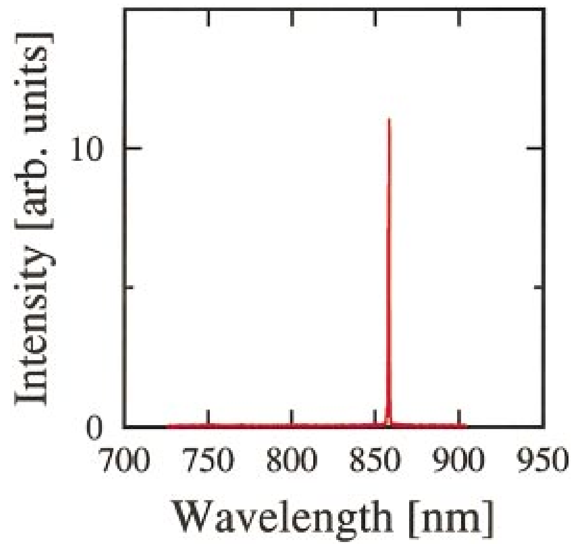

FIG. 4. (Color) Lasing spectrum in the case of 10 mw output power of the closed unstable resonator.

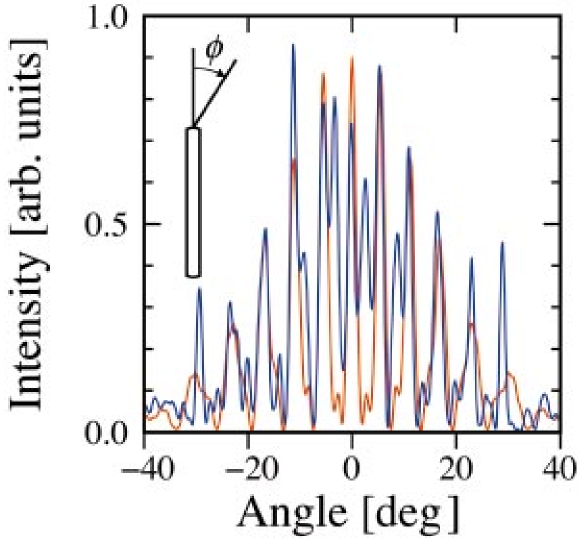

FIG. 5. (Color) Far-field patterns of the closed unstable resonator. The red curve is obtained numerically and the blue one observed in the experiment.

The lasing mechanism of the optical resonators we report in this paper is understood intuitively as follows. If the angle of reflection on the side wall mirrors are larger than the critical angle $\theta(\sin\theta=1/3.3)$, the light can be confined in the resonator due to total internal reflection, and grow exponentially while propagating in the longitudinal direction so that it gains the pumping energy from the lasing medium. When the growing speed of the light intensity exceeds the cavity loss, the resonator can lase.

Accordingly, the lasing mechanism of the laser diode we present here is very similar to the conventional unstable resonators in the sense that the light grows when it propagates in the longitudinal direction [19]. However, the conventional unstable resonators do not have side mirrors, and hence they have only one UPO. On the other hand, there exist infinite UPO's in the closed unstable and concentric resonators. Therefore, we cannot expect one to one correspondence between the resonator eigenmodes and UPO's in the cases of the closed unstable and concentric resonators [1].

First, let us discuss the closed unstable resonator, of which cavity length and width are 700 μm and 60 μm, respectively, while the radius of curvature of the circular end mirror is 60 μm. The resonance eigenfunction of the Helmholtz equation for an unstable resonator corresponding to the wavelength $2\pi/k=855.7$ nm outside the resonator is approximately calculated by the above method, and shown in Fig. 2(a). The refractive index n inside the resonator is assumed to be 3.3. One can easily recognize the scar, i.e., the localization of the light intensity on a few UPO's in Fig. 2(b). If this mode can lase, the far-field pattern has the signature characteristic of this scar wave function due to the coherence of the laser light.

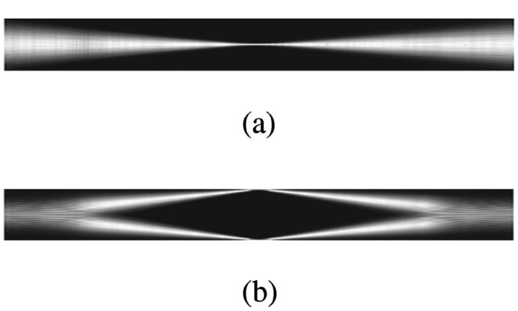

FIG. 6. The eigenmodes of the closed concentric resonator; (a) mode 0, (b) mode 1.

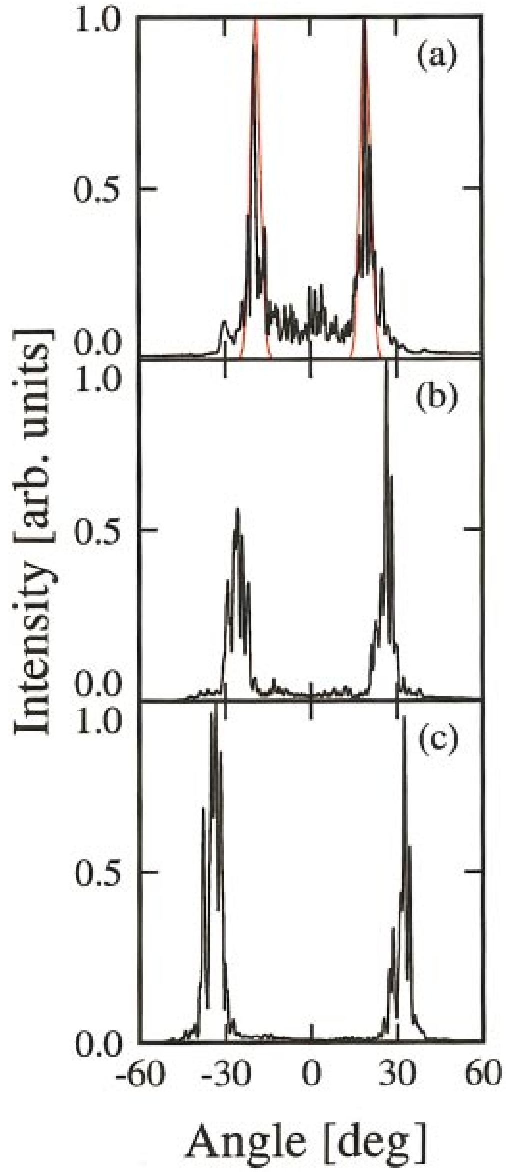

FIG. 7. (Color) The far-field patterns of the scar eigenmodes of the closed concentric resonator. The width of the resonator is (a) 60 μm, (b) 80 μm, and (c) 100 μm. The red curve in (a) is the calculated far-field pattern corresponding to the scar eigenmode of Fig. 6(b).

Now we show the experimental results of the laser diode of the unstable resonator shape is tested at 25 °C using a pulsed current with 500-ns width at 1-kHz repetition. The relation between the injection current and the intensity of the output light is shown in Fig. 3. The threshold phenomena can be clearly observed, that is, the laser action of the microcavity laser diode. Figure 4 is the spectrum of the output light from the laser diode of the unstable resonator shape. The sharp peaks clearly show the laser action.

The red curve of Fig. 5 is the calculated far-field pattern corresponding to the scar wave function in Fig. 2(a), while the blue curve is the result obtained from the actual experiment. One can see the nice correspondence between both structures. Consequently, we conculde that we observed the laser action on the scar mode of Fig. 2. The wave function localizes on a few UPO's inside the resonant cavity as shown in Fig. 2. However, the beams are so diffusive that they interfere with one another and the periodic orbit structures disappear in the far-field pattern of the closed unstable resonator.

Next, let us move to the closed concentric resonators, of which cavity length and width are 600 μm and 60 μm, respectively, while the radius of curvature of the circular end mirror is 300 μm. The conventional concentric resonators have been considered in the research field of the laser resonators as a precisely neutral case of stability because the ray-dynamical periodic orbits bouncing only circular mirrors are neither stable nor unstable [19]. However, the closed concentric resonators we discuss here have the mirrors on the side walls, which do not affect the neutral periodic orbits at all, but produce infinite UPO's by multiple scattering, and ensure full chaoticity of the ray dynamics, as exactly proven by Bunimovich [20].

The quasieigenstates of the Helmholtz equation for a closed concentric resonator are obtained numerically by the extended Fox-Li mode calculation method explained above, and shown in Fig. 6. The least loss mode 0 corresponds to the neutral periodic orbit. Accordingly, mode 0 is not a scar wave function, but mode 1 that has larger loss than mode 0 is a scar wave function localizing on the "diamond" shape UPO. Unfortunately, one cannot expect that mode 1 would lase because mode 0 overcomes mode 1 due to the small loss. Indeed, we observed laser action on mode 0 by the far-field pattern, which has one strong peak corresponding to mode 0.

We changed the region of current injection from the whole cavity into the diamond region corresponding to mode 1 by forming the diamond-shape p electrode. Then we observed the laser action on mode 1 that has two peaks in the far-field pattern, nicely corresponding to the calculated one, as shown in Fig. 7(a). The spacings of the beams corresponding to diamond-shape UPO's are so wide that the beams do not intefere with one another even far outside the resonator, and so the periodic orbit structures are observed as the strong peaks of the far-field pattern.

When we vary the width of the closed concentric resonator from 60 μm to 80 and 100 μm with the same length of the resonator and radius of curvatures of the curved end mirrors, the positions of two peaks of the far-field pattern change as shown in Figs. 7(b) and 7(c) because the shape of diamond-shape UPO's also vary. The peak positions of the far-field patterns of the closed concentric resonator, of which widths are 60, 80, and 100 μm, respectively, calculated by applying the Snell's law to the diamond-shape UPO's are $\pm 19.2°$, $\pm 25.9°$, and $\pm 32.9°$. One can see that the peaks calculated by the Snell's law excellently correspond to the peaks of the observed far-field patterns as shown in Fig. 7.

In summary, we numerically obtained the scar eigenmodes of the closed unstable and concentric resonators. The far-field patterns corresponding to the scar eigenmodes were observed in the experiments of the semiconductor microlasers. The laser action on the scar modes were controlled by the shape of the electrode for the current injection.

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